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## Wilsonian vs. 1PI renormalization group flow irreversibility

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### Abstract

We present a line of reasoning based on the analysis of scale variations of the Wilsonian partition function and the trace of the stress tensor in a curved manifold which results in a statement of irreversibility of Wilsonian renormalization group flow for unitary theories. We also analyze subtleties related to subtractions in the case of the 1PI effective action flow.

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## Note

An error in section 2 (which does not affect sections 3 and 4) invalidates the flow equation for  $c_W$ . The paper has been temporarily withdrawn from publication.

## 1 A bit of history

Loss of information seems inevitable as short-distance degrees of freedom are integrated out in favor of an effective long-distance description of any physical system. This intuition, which amounts to a statement of irreversibility of renormalization group (RG) trajectories, might be deceptive. A renormalization group transformation is made out of two steps: integration of modes (Kadanoff transformation) followed by a rescaling of the variables of the system. Their combination is not obviously irreversible, in particular when a reshuffling of the Hilbert space comes along with the long distance realization of the theory.

The efforts to sort out this question started with the perturbative analysis of  $\lambda\phi^4$  with  $N$ -components made by Wallace and Zhia [1], proving that the beta functions of the model are indeed gradient flows up to three loops. The irreversibility conjecture was proven to be correct in that context and, furthermore, it was immediate to realize that

$$\beta_i = \partial_i c \quad \longrightarrow \quad \int_{g_{UV}^*}^{g_{IR}^*} dg^i \beta_i = \Delta c \geq 0, \quad (1)$$

where  $c = c(g^i)$  is the function from which  $\beta_i$  are derived and  $g_{UV}^*, g_{IR}^*$  label the UV and IR fixed points delimiting the RG flow. Thus, a sufficient, but not necessary, condition for irreversibility of RG flows is that the  $\beta$ -functions of a theory are gradient flows. What is just necessary to prove is that an observable quantity existing in any theory decreases monotonically along the RG flows.

With the advent of conformal field theories, Zamolodchikov constructed a function that decreases along RG trajectories in two-dimensional unitary field theories [2]. This function reduces to the central charge of the conformal theory at fixed points (thus, Zamolodchikov's result is often referred as the c-theorem). The elements of Zamolodchikov's proof were Lorentz invariance, conservation of the stress tensor and unitarity.

Later on, some groups have forcefully tried to extend Zamolodchikov's remarkable achievement to higher dimensional theories. Cardy [3] considered the flow of the integrated trace of the stress tensor on a sphere,  $S^n$ , as a candidate of a c-function. His idea consisted in trading the explicit subtraction point dependence ( $\mu$ ) for the one in the radius of the sphere ( $a^{-1}$ ),

$$c(g(t), t) \equiv \langle \int_{S^n} \sqrt{g(x)} \theta(x) \rangle, \quad (2)$$

where  $t \equiv \ln \frac{\mu}{a}$ , so that  $\mu \partial_\mu = -a \partial_a$ , where  $a$  is the inverse of the radius of the sphere. A RG transformation, usually understood as an enlargement of all relative distances, is here realized as a blow up of the sphere. The RG flow of this c-candidate is related to the correlator of two stress tensors, yet the appearance of other contributions seemed to spoil the irreversibility proof. An appealing feature of Cardy's idea is that his candidate reduces to the Euler density coefficient of the trace anomaly at conformal field theory [4]. For instance, in four dimensions,

$$\langle \theta(x) \rangle_{cft} = \frac{1}{2880} (-3aF(x) + bG(x) + c\Box R(x)) \implies \langle \int \sqrt{g} \theta \rangle_{cft} = \frac{1}{2880} 32\pi^2 b\chi(M), \quad (3)$$

where  $F = C_{\mu\nu\rho\sigma}^2 = R_{\mu\nu\rho\sigma}^2 - 2R_{\mu\nu}^2 + \frac{1}{3}R^2$ ,  $G = R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2$  and  $\chi(M)$  the Euler characteristic of the manifold. Whereas  $a$  has been proven to be related to the positive coefficient of the spin 2 structure in the correlator of two stress tensors in flat space [5],  $b$  is only empirically known to be positive for conformal free bosons, fermions and vectors (its actual value is 1, 11 and 62, respectively). This  $b$  coefficient as a candidate was further analyzed and modified by Osborn [6] proving an evolution equation reminiscent of the two-dimensional case. Unitarity did not enter the construction and, again, irreversibility was not proven.

A different approach was taken in ref. [5, 7] where the starting point was unitarity via the spectral representation of two stress tensor correlators. A reformulation of Zamolodchikov's result is achieved considering

$$\langle T_{\alpha\beta}(x)T_{\rho\sigma}(0) \rangle = \frac{\pi}{3} \int_0^\infty d\nu \, c(\nu, t) \int \frac{d^2p}{(2\pi)^2} e^{ipx} \frac{(g_{\alpha\beta}p^2 - p_\alpha p_\beta)(g_{\rho\sigma}p^2 - p_\rho p_\sigma)}{p^2 + \nu^2} \quad . \quad (4)$$

The spectral function describes the central charge of UV and IR fixed points associated to a given flow as

$$c(\nu, t) = c_{IR} \delta(\nu) + c_{smooth}(\nu, t) \quad c_{UV} = \int d\nu \, c(\nu, t) \quad (5)$$

where  $t$  is the RG flow parameter. Therefore,  $c_{UV} \geq c_{IR}$ . At long distances, only massless modes survive, entering the spectral representation as a delta of the spectral parameter. Unitarity yields positivity of  $c_{smooth}$ , thus, irreversibility. This proof emphasizes the role of the stress tensor as a mean to account for all degrees of freedom, and to quantify the decoupling of massive ones along RG trajectories. The extension of this idea to higher dimensions encounters a first complication due to the existence of two spin structures in the  $\langle TT \rangle$  correlator. Since RG flows are related to a change of scale, it is natural to focus attention in the spin 0 spectral density. The problem turns out to be that, at conformal field theories in  $n$  dimensions,

$$c^{(0)}(\nu) \longrightarrow c^{(0)}\nu^{n-2}\delta(\nu) \quad (6)$$

and  $c^{(0)}$  is unobservable, at least as a well-defined quantity of the conformal field theory. The way out proposed was to define  $c^{(0)}$  using a limit from the flow itself

$$c^{(0)}\delta(\nu) \equiv \lim_{t \rightarrow 0} \frac{c^{(0)}(\nu, t)}{\nu^{n-2}} \quad . \quad (7)$$

Any limiting procedure which allows for a definition of  $c^{(0)}$  yields a decreasing quantity. The c-theorem is then proved in any perturbative expansion. These results were verified through examples in ref. [7].

Further efforts have been devoted to understand the validity of the above proposed candidates as well as to formulate the theorem by other means [8]. Let us here summarize what we think are essential ingredients of the statement of irreversibility of RG flows and, consequently, ought to enter its proof in one way or another. We reduce such essentials to two:

a) The c-function must be sensitive to all degrees of freedom of the theory.

b) The sign of the derivative of the c-function with respect to the flow parameter must be dictated by unitarity.

Therefore, it is not essential that the c-function reduces to any simple quantity known in the theory, although it would be welcome that such a possibility were realized. Neither much emphasis should be put on the fact that beta-functions are gradient of the c-function.

Upon a simple reflection, it is clear that all candidates so far explored, including Zamolodchikov's original one in two dimensions, are constructed from the stress tensor. This operator exists for every theory and couples to *all* degrees of freedom. Furthermore, it does not pick anomalous dimensions and its correlators obey useful Ward identities. It is not yet settled whether the attempts to formulate the theorem in higher dimensions have so far missed some basic ingredient. On one hand, the integrated trace of the energy momentum tensor, which carries spin 0, generates a change of scale, but its variation has not been related to unitarity. On the other hand, the spectral representation approach lacks the explicit formulation of a candidate at the conformal theory. Yet the above requisits may be too stringent. The lesson from two dimensions is that  $c$  counts massless degrees of freedom whereas  $\Delta c$  quantifies the decoupled massive modes. This modifies a little bit the above point a). Similarly, we may also bring down unitarity requirements of point b) in the following way. If the  $c$ -function is stationary at fixed points, it is just sufficient to find a negative sign of its derivative anywhere along the flow. Then, the function will remain decreasing till a new fixed point is reached.

We here propose to combine all this accumulated knowledge in the following way. There is an obvious quantity in any field theory which is sensitive to all degrees of freedom: the Wilsonian partition function. We define it (in euclidean space) as

$$Z[g^i(t), t] \equiv \int \prod_{\Lambda_{IR} \leq p \leq \Lambda_{UV}} d\varphi_p e^{-S}, \quad (8)$$

where  $t \equiv \ln \frac{\Lambda_{UV}}{\Lambda_{IR}}$  is the RG flow parameter. As more degrees of freedom are integrated out,  $t \rightarrow \infty$ . Often, we consider field theories with the purpose of computing Green functions. Then, the renormalization procedure trades  $\Lambda_{UV}$  for a subtraction point  $\mu$  and  $\Lambda_{IR}$  can be safely sent to 0 if external momenta are different from zero (otherwise, an IR regulator is necessary). We shall come back to this generating function for connected Green functions later on.

The absence of external sources shields the properties of the partition function from a simple analysis. We have found most convenient to consider field theories defined on a curved space, *e.g.* on the  $S^n$  sphere with radius  $a^{-1}$ . We may consider  $a$  to be arbitrarily small, so that we use it both as an external source for the stress tensor and as an IR regulator, taking over the role of  $\Lambda_{IR}$ . Stress tensor correlators are defined as [5],

$$\langle \sqrt{g} T^{\mu\nu}(x) \rangle = -2V \frac{\partial \ln Z}{\partial g_{\mu\nu}(x)}, \quad (9)$$

$$\langle \sqrt{g} T^{\mu\nu}(x) \sqrt{g} T^{\alpha\beta}(y) \rangle = 4V^2 \frac{\partial}{\partial g_{\mu\nu}(x)} \frac{\partial}{\partial g_{\alpha\beta}(y)} \ln Z + \frac{V}{n} \langle \theta \rangle \delta^n(x-y) \sqrt{g} \left( g^{\mu\nu} g^{\alpha\beta} - g^{\nu(\alpha} g^{\mu\beta)} \right), \quad (10)$$

where  $V = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})}$  is the standard volume factor of a sphere. A simple definition of these correlators in terms of functional derivatives with respect to the metric would lead to a violation of diffeomorphism Ward Identities at contact terms. This is the reason to subtract a delta term in eq. (10), which has been further simplified using properties of maximally symmetric spaces. As it stands, both  $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$  and  $\nabla_\mu \langle T^{\mu\nu} T^{\alpha\beta} \rangle = 0$  are obeyed. Further properties of  $T^{\mu\nu}$  are discussed in ref. [9, 10].

We are now in the position of streamlining our construction.

## 2 Irreversibility of the Wilsonian RG

Consider a field theory defined on a  $S^n$  sphere of radius  $a^{-1}$ . We construct the following dimensionless quantity based on the Wilsonian partition function

$$c_W(g^i(t), t) = \int \sqrt{g(x)} \langle \theta(x) \rangle + nV \ln Z, \quad (11)$$

where  $t = \ln \frac{\Lambda}{a}$ ,  $\Lambda$  being an UV scale. This function obeys the RG equation

$$\left( \frac{\partial}{\partial t} + \beta_i \frac{\partial}{\partial g^i} \right) c_W(g^i(t), t) = 0. \quad (12)$$

We need to study the RG flow of  $c_W$ , that is

$$\frac{\partial}{\partial t} c_W = -\beta_i \frac{\partial}{\partial g^i} c_W. \quad (13)$$

The  $t$  dependence can be computed as

$$\frac{\partial}{\partial t} = -a \frac{\partial}{\partial a}, \quad (14)$$

which reflects the fact that changing the scale at which physics is considered can be done by varying the radius of the sphere. This is nothing else than the starting point of Cardy's analysis. In a symmetric space it is also true that

$$a \frac{\partial}{\partial a} = -2 \int g_{\mu\nu}(x) \frac{\delta}{\delta g_{\mu\nu}(x)}, \quad (15)$$

a change in the scale factor is obtained as a Weyl transformation.

Using the definitions in eq. (9, 10) we, first, have that

$$\sqrt{g(x)} \langle \theta(x) \rangle = -2V g_{\mu\nu}(x) \frac{\delta}{\delta g_{\mu\nu}(x)} \ln Z, \quad (16)$$

and, second,

$$\sqrt{g(x)} \langle \theta(x) \theta(y) \rangle = -2V g_{\mu\nu}(x) \frac{\delta}{\delta g_{\mu\nu}(x)} \langle \theta(y) \rangle. \quad (17)$$

As a particular case, these relations control the trace anomaly at conformal field theory. The trace of the energy momentum tensor does not vanish, neither its correlators at coincident points, although  $\beta_i(g^{j*}) = 0$ . Putting together all the ingredients

$$\frac{\partial}{\partial t} c_W = -a \frac{\partial}{\partial a} c_W = 2 \int g_{\mu\nu}(x) \frac{\delta}{\delta g_{\mu\nu}(x)} c = -\frac{1}{V} \int_x \int_y \sqrt{g(x)} \sqrt{g(y)} \langle \theta(x) \theta(y) \rangle \quad (18)$$

Thus,

$$\frac{\partial}{\partial t} c_W = -\frac{1}{V} \langle \int_x \sqrt{g} \theta(x) \int_y \sqrt{g} \theta(y) \rangle \leq 0. \quad (19)$$

The c-function we have constructed from the Wilsonian partition function decreases along RG flows. All intermediate steps are well-defined as the partition function is equipped with both an UV and

IR cut-off at the outset. This obviates problems related to subtractions or non-analiticity at zero momentum, in close analogy to the currently widely used idea that the Wilsonian effective action is analytic in momenta. The exact renormalization group equation can be expanded in momenta due to the ubiquitous presence of cut-offs. It is a delicate issue how to handle the case of the flow for the 1PI effective action, which we postpone to the next section.

Our candidate in eq. (11) provides a synthesis of several ideas previously considered. It takes Cardy's conjecture as a starting point, bringing a piece of the needed irreversibility together with a term which spoils it. This is corrected with the introduction of the log of the partition function. Some authors had conjectured that the partition function itself would be enough to provide a c-function. Such a possibility is definitely ruled out in flat space as easily seen in the example of a free boson plus a free fermion. The overall partition function cancels, therefore the piece coming from a free boson flows decreases as the one for a free fermion increases. It also follows that a naive use of the exact renormalization group equation to settle irreversibility of the flow is insufficient. The equation controls the flow of the partition function which does not decrease monotonically as we just pointed out.

### 3 1PI effective action flow

We would like to explore the limit of the flow equation we have gotten in the previous section to the case where  $\Lambda_{UV}$  is sent to infinity. This is carried through a standard renormalization procedure that trades  $\Lambda_{UV}$  for a subtraction point  $\mu$ , often sending also  $\Lambda_{IR} \rightarrow 0$ , when external momenta of Green functions are kept different from zero. This limit of the partition function leads, upon a legendre transformation, to the 1PI effective action. For convenience, we shall call it 1PI partition function. The Wilsonian partition function flow interpolates between two theories, but loops associated to the second, IR, theory remain to be done. In the 1PI effective action we do integrate all modes.

From the point of view of our irreversibility argument, two issues need to be reconsidered. The first one,  $\Lambda_{IR} \rightarrow 0$  is bypassed since the curvature provides a natural IR cut-off. To keep  $b$  as an essential ingredient of the c-function at conformal points, it is necessary to work in curved space. Therefore, we concentrate on the UV limit.

Let us first note that the stress tensors correlators are defined from the renormalized partition function. Standard coupling constants and wave-function renormalizations have been performed. Furthermore, the stress tensor combines the wave-function renormalization of its constituent fields with a composite operator one such that  $\theta_{bare} = \theta_{ren}$  and, thus, carries no anomalous dimensions. What remains is just a subtraction in the two-point correlator  $\langle T_{\mu\nu}(x) T_{\alpha\beta}(0) \rangle$ . It is convenient to understand this point in terms of the freedom of scheme brought by the renormalization procedure. In four dimensions, the above correlator has dimension 8, but Ward Identities dictate the presence of 4 derivatives which leaves a freedom of a contact term.

On a sphere of radius  $a^{-1}$ , the spectral representation takes the form

$$\langle \theta(x) \theta(0) \rangle = \frac{\pi^2}{40} a^2 \int_{\frac{5}{2}}^{\infty} d\sigma \rho(\sigma) \left( \Delta - 4a^2 \right)^2 G(\sigma, r), \quad (20)$$

where  $\sigma$  labels the scalar representations of  $SO(1, 4)$ ,  $\Delta$  is the covariant laplacian,  $r$  the geodesic distance and  $G(\sigma, r)$  the appropriate Green function,  $\left[ \Delta - a^2(\sigma^2 - \frac{9}{4}) \right] G(\sigma, r) = \frac{\delta^4(x)}{\sqrt{g(x)}}$ . At CFT,

$\rho(\sigma) = \rho_0 \delta(\sigma - \frac{5}{2})$ . A detailed analysis [5] reveals the presence of a contact term

$$\langle \theta(x) \theta(y) \rangle = \frac{\pi^2}{40} a^2 \rho_c \left( \Delta - 4a^2 \right) \frac{\delta^4(x-y)}{\sqrt{g(x)}}. \quad (21)$$

Using eq. 17, one proves that this contact term is related to the coefficient of the spectral function at the conformal point,  $\rho_0 \geq 0$ , and the  $b$  trace anomaly coefficient in the following way [5]

$$b = \rho_0 + \rho_c. \quad (22)$$

Through examples,  $\rho_c$  at most cancels  $\rho_0$  (in odd dimensions) but never overcomes it, leaving  $b \geq 0$  always. We have a weak argument for this result. Take a resolution of the identity on the r.h.s. of (19),

$$- \sum_n \langle \int \sqrt{g} \theta | n \rangle \langle n | \int \sqrt{g} \theta \rangle \leq 0. \quad (23)$$

A unitarity argument of this sort, if applicable on integrated objects, yields irreversibility for the 1PI effective action. The above argument is protected from IR infinities in  $S^4$ . This reasoning would preserve the sign of the r.h.s. in eq. (19) in the 1PI case at the same time that would also explain the elusive positivity of  $b$ .

## 4 Review of Cardy's proposal

As discussed earlier, a theorem stating irreversibility of RG trajectories just needs proving a definite sign for the derivative of an observable quantity on the flow parameter. This, though, stays one step short from a powerful quantitative tool if there is no simple way to characterize the c-function. The discussion in the two previous sections is missing such a point. We already noted that the welcome properties of a c-function are those offered by the original two-dimensional case: the c-function should be stationary at conformal field theory; at these points, the c-number should be easily computable; and, in the best of the worlds, beta-functions should be gradient flows.

The understanding of contact terms in stress tensor correlators, as explored in the previous section, allows for a more detailed analysis of a simpler candidate for the c-theorem. Let us reconsider Cardy's proposal of eq. (2) in dimension 4. Its variation along the flow can be derived using the equations in section 2,

$$\frac{\partial}{\partial t} c(g^i(t), t) = -\frac{1}{V} \int_x \int_y \sqrt{g(x)} \sqrt{g(y)} \langle \theta(x) \theta(y) \rangle + n c(g^i(t), t). \quad (24)$$

This quantity is not obviously negative due to the presence of the last term. One can trace the two terms in the r.h.s. to the variation of  $\theta$  and  $\sqrt{g}$  respectively. In a way, the competition of these two terms is forced by the adimensional character of  $c$ . At conformal field theory, this c-function reduces to the  $b$  coefficient of the trace anomaly and is stationary,

$$\left. \frac{\partial}{\partial t} c \right|_{g^i = g_*^i} = \frac{\partial}{\partial t} b = 0, \quad (25)$$

because the r.h.s. yields an exact cancellation between the contact term coming from the two-point correlator and the trace anomaly, as derived from eq. (17).

It is clear that  $\partial_t c = 0$  at fixed points implies that it is sufficient to find a negative sign of  $\partial_t c$  at any intermediate point to have a proof of irreversibility. This, indeed, was done by Cardy near a fixed point using conformal perturbation theory for quasi-marginal deformations. His computation builds more confidence of the validity of the theorem but is still not a proof.

Let us go back to the above observation about the exact cancellation of the r.h.s. of eq. (24) at fixed points. The origin of this simplification is rooted in the operator product expansion of  $\theta(x)\theta(0)$ . It is known that the identity contribution in the flat space OPE is [4, 5, 10]

$$T_{\mu\nu}(x)T_{\alpha\beta}(0) \sim c^{(2)}\Pi_{\mu\nu,\alpha\beta}^{(2)}(\partial)\frac{1}{x^4} + \dots \quad (26)$$

where  $\Pi_{\mu\nu,\alpha\beta}^{(2)}(\partial)$  stands for a spin 2 projector and  $c^{(2)}$  can be shown to coincide with the  $a$  coefficient of the Weyl square density in the trace anomaly (3). This, indeed, provides a connection between spin 2 flat space non-local correlators and spin 0 contact terms in curved space. In curved space, the OPE for the traces of stress tensors contains a delta term

$$\theta(x)\theta(0) \sim \frac{4}{V}\theta(0)\delta^4(x) + \text{non-local terms} \quad (27)$$

Indeed, this contact term is needed in the OPE for consistency of the anomaly in curved space. The factor 4 is related to the classical (as well as quantum) dimension of  $\theta$ , whereas the factor  $V$  is present due to our definition of  $\theta$ . No other contact terms are present by dimensional and scaling arguments. The global prefactor is fixed by the way the OPE works in the conformal case. Away from the fixed point, we have so far no control on the relation between the two terms in the r.h.s. of eq.(24).

The above analysis adds some understanding to the RG flow of the Euler density coefficient in the trace anomaly but does not prove its irreversibility yet. More inconclusive but tantalizing evidence was presented through an example by Cardy. Consider QCD at short and long distances. Asymptotic freedom allows for an easy computation of  $c_{UV} = 11N_f N_c + 62(N_c^2 - 1)$  whereas its chiral realization implies  $c_{IR} = (N_f^2 - 1)$ , since  $b = 1, 11, 62$  for bosons, fermions and vectors. It follows that  $c_{UV} > c_{IR}$ . Moreover, in ref. [11], a large number of exact results have been checked against the above c-function and systematic validity of the would-be theorem has been found.

One deep, uncanny lesson hidden in this ideas is that fermions *weight* more than bosons. This is not so in two dimensions as a Dirac fermion has the same central charge as a boson, which is at the origin of exact bosonization. This is no longer true in higher dimensions. Long distance realizations would, in general, favor bosons. This might represent just a glimpse of a deep relation between RG irreversibility and Goldstone theorem.

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